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THE SIMILARITY HYPOTHESIS APPLIED TO TURBULENT FLOW IN AN ANNULUS

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Abstract—The velocity distribution in fully developed turbulent flow in an annulus is investigated using Goldstein's [1] extension of the similarity hypothesis of von Kármán. The theoretical result is compared with the experimental data of various investigations. The validity of the similarity theory in the study of turbulent flow in an annulus is examined using the measurements of the turbulence characteristics of Brighton and Jones [2]. The results indicate that the adoption of the theory is justified in the outer region of the flow only, i.e. outside the radius of maximum velocity. A particular form of the theory, which was found [1] to predict the velocity distribution accurately in the simpler pipe flow geometry, is used in the annulus study. The analysis of the annular flow is such that the computed results for the pipe can then be used to advantage.

The simpler correlations for the velocity distribution in the form of the "law of the wall" are briefly discussed, and the inadequacy of these in the inner region of the flow, i.e. inside the radius of maximum velocity, is pointed out. For completeness, a simple modified equation of a semi-empirical nature is presented. The final result for the inner region is compatible with the results for the limits of the annular geometry, viz. the pipe and the parallel wall channel. It is noted that the velocity distribution in the outer region of the annulus is accurately described using the simple logarithmic laws.

NOMENCLATURE

$b, b_1, b_2, A, B, k, k_1, k_2,$	constants;
l,	mixing length, or characteristic length;
u, v, w,	velocities (u in the direction of the mean flow);
u_1, v_1, w_1,	fluctuations of velocities;
$u_\tau = \sqrt{(\tau_w/\rho)}$,	friction velocity;
$u^+ = u/u_\tau$,	friction velocity parameter;
$y^+ = yu_\tau/\nu$,	friction distance parameter;
τ,	shear stress;
r,	radius;
$\eta = r/r_0$, $\theta = r/r_2$, $\psi = r/r_1$,	dimensionless radii;
$a = r_2/r_1$,	radius ratio;
ρ,	density;
ν,	kinematic viscosity;
f_1, f_2,	functions;
y,	distance from a wall.

Suffixes

w,	wall;
1,	inside radius of maximum velocity, r_m, or inner wall;
2,	outside radius of maximum velocity, r_m, or outer wall;
m,	point of maximum velocity;
0,	value of the outer radius in pipe flow.

Superscripts

A single dot denotes first differential coefficient with respect to radius.

A double dot denotes second differential coefficient with respect to radius.

1. INTRODUCTION

THE LAWS of the velocity distribution in turbulent flow adjacent to a solid boundary and in ducts are essentially of a semi-empirical nature. The ideas, concepts and theories are well known, and are recorded in standard works in fluid mechanics and heat transfer.

A major contribution in this field of study is the similarity theory of von Kármán. In the present work, Goldstein's [1] extension of Kármán's theory to axisymmetric flow is of special interest and it suffices here to present the

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results of this work. According to Goldstein [1], for three dimensional turbulence, the characteristic length, the stress and the rate of transfer of $(\tau . r)$ are respectively given by

$$\left.\begin{aligned} &\text{(i)}\ l = k\dot{u}/(\ddot{u} - \dot{u}/r) \\ &\text{(ii)}\ |\tau| = \rho l^2 \ddot{u}^2 \\ \text{and}\quad &\text{(iii)}\ \frac{1}{r}\frac{\partial}{\partial r}(\tau r) = \rho l^2 \dot{u} . (\ddot{u} - \dot{u}/r) \end{aligned}\right\} \quad (1)$$

We pay particular attention to (iii) of equation (1), because this leads to an equation which shows excellent agreement with the observed velocity data in pipe flows. The final result for the velocity is given by Goldstein [1], as

$$\left(\frac{u_m - u}{u_\tau}\right) = \frac{\sqrt{3}}{k} \int_0^{r/r_0} \frac{\eta . d\eta}{(1 - \eta^3)^{\frac{1}{2}}} + b \qquad (2)$$

where $\eta = r/r_0$, and the constants k and b are chosen so as to make the equation suit the experimental data over the region where the hypothesis is most likely to be valid.

In this work, we will examine the velocity distribution in axisymmetric turbulent flow in an annulus. Goldstein's theory will be employed because it accurately predicts the velocity distribution in axisymmetric flow in a pipe.

2. VELOCITY DISTRIBUTION OF TURBULENT FLOW IN AN ANNULUS

A brief outline of some of the methods of correlation of turbulent flow in an annulus has been given by Barrow [3].

Probably the simplest approach which is used, is to employ the familiar logarithmic law for zero pressure gradient, that is:

$$u^+ = (1/k) \ln y^+ + B \qquad (3)$$

According to Goldstein [4], this equation might be expected to apply to pressure flows to a first approximation. In the annulus, equation (3) has been employed with the same constants as those used in the pipe. For example, Deissler and Taylor [5] have (used $k = 0{\cdot}36$ and $B = 3{\cdot}8$ for the regions inside and outside the radius of maximum velocity, but in the light of recent reliable measurements [6] it is debatable whether or not a single form of the equation is valid.

A further difficulty exists in the case of turbulent annular flow. The position of zero shear is unknown and as a consequence, the wall stresses cannot be determined. It is sometimes assumed that the radii of zero shear and maximum velocity and the radius given by Lamb [7] are coincident. In Section 5, the experimental data is evaluated using the actual radius of maximum velocity; the theoretical work is based on Lamb's radius. Little error is involved in this procedure provided α is not much larger than 3. In both cases, zero stress is assumed to occur at the radius of maximum velocity.

It would appear that an equation of the form,

$$u^+ = f_1(\alpha) \cdot \frac{1}{k} \ln y^+ + f_2(\alpha) . B \qquad (4)$$

would be more in keeping with the pipe result, but an examination of the experimental data suggests that modification of the pipe equation is necessary only in the region inside r_m, the values of $f_1(\alpha)$ and $f_2(\alpha)$ being given in Appendix 1.

Rothfus, Monrad and Senecal [9] have used equation (3) for the annulus, the "wall distance" being a complex function of the actual wall distance. With $u^+ = u/u_{\tau 2}$, the equation was found to be valid both inside and outside r_m and more recent data [6] are found to be in support of this correlation.

It can be seen that attempts to correlate velocity distribution in the annulus have been largely influenced by the well established results for the pipe. In some respects this has tended to suppress more detailed investigations which have their origin in the study of the fundamentals of the flow. In the following section, the velocity in turbulent flow in the annulus is studied using one set of the basic results of the similarity hypothesis according to Goldstein [1]. It is first necessary to ascertain that the assumptions made in the theory are adequately fulfilled by the flow conditions in the annulus. In this connection, the turbulence characteristics in the annulus will be examined along with those for the simpler pipe flow. This might be of some help in assessing the accuracy of the final result.

3. TURBULENCE CHARACTERISTICS

It appears that little has been done in the measurement of turbulence characteristics in annular flows. Probably the most recent data on velocity fluctuations in turbulent flow in an annulus is that of Brighton and Jones [2]. Some of their data has been compiled in a form which is more suitable for present purposes, and is shown in Fig. 1, along with some results of Laufer for pipe flow [10], (see also reference 8).

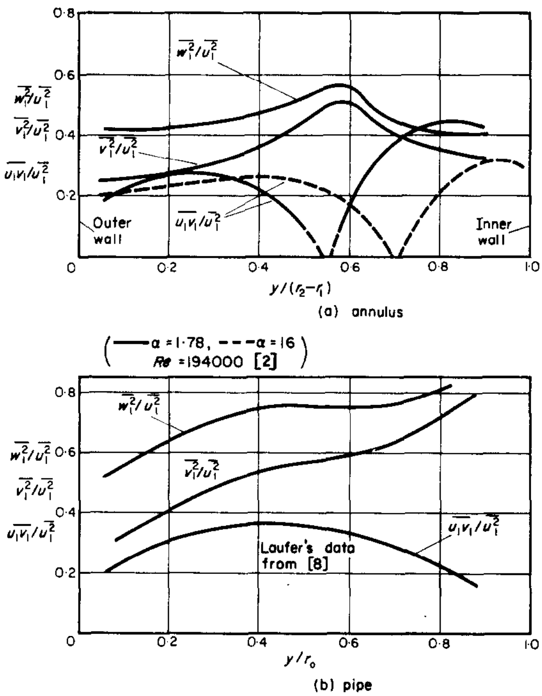

FIG. 1. Turbulence characteristics.

We note that in the similarity theory, the ratios $\overline{u_1^2}:\overline{v_1^2}:\overline{w_1^2}:\overline{u_1v_1}:\overline{v_1w_1}:\overline{u_1w_1}$, should be constant. This is clearly not so in either flow geometry, the greatest departure from the assumption occuring in the region of zero stress and near the wall where for example the value of $\overline{u_1v_1}$ approaches zero. In these regions therefore, we must not expect good agreement with theory in either geometry. The most interesting feature of the data for the annulus geometry is the variation in the quantity $\overline{u_1v_1}/\overline{u_1^2}$ across the flow section and its dependency on the radius ratio, α. In the outer region $\overline{u_1v_1}/\overline{u_1^2}$ is fairly constant, whereas inside the radius of maximum velocity there is a noticeable dependency of the value of this ratio on both position and radius ratio. Brighton and Jones [2] have already observed that the pronounced curvature of the curve of $\overline{u_1v_1}/u_{\tau 2}^2$ in the inner region of an annulus is accompanied by a marked departure of the velocity distribution from the law of the wall. The large variation of $\overline{u_1v_1}/\overline{u_1^2}$ in the inner region must also lead to large deviations from a prediction based on the similarity hypothesis which assumes a constant value for this ratio. It can be clearly seen by comparing Figs. 1(a) and (b), that the magnitude and distribution of the turbulence characteristics in the pipe and annuli are not markedly different outside r_m, but inside r_m there is poor agreement particularly as far as the ratio involving $\overline{u_1v_1}$ is concerned.

It is concluded therefore, that the assumption concerning the ratios $\overline{u_1^2}$ etc. is satisfactorily met with in the outer region of an annulus while inside the radius of maximum velocity the situation is far from the idealised one. It will be seen later, when the theoretical and experimental velocity distributions are compared, that there is excellent agreement in the outer boundary layer. The agreement in the inner region is however disappointing.

With these observations on the turbulence characteristics in mind, we now proceed to derive a velocity law for turbulent flow in an annulus. It is felt that the observations on the similarity between the turbulence parameters in the pipe and annulus provide adequate justification for using similar approaches for the determination of velocity distributions.

This is certainly true in the outer area; in the inside region of an annulus, the procedure is questionable and less likely to yield a good correlation.

4. THEORETICAL ANALYSIS

(i) *Velocity distribution outside the radius of maximum velocity*

The shear stress, τ, in an annulus is written,

$$\tau = \tau_{w1} \cdot \frac{r_1}{r} \cdot \left[\frac{r^2 - r_m^2}{r_m^2 - r_1^2}\right] \ (r_2 > r > r_m). \qquad (5)$$

Equation (5) is easily derived from momentum considerations assuming that zero stress occurs at r_m. With the expression for l, equation (1) (iii) then becomes

$$\left(\frac{\ddot{u} - \dot{u}/r}{\dot{u}^3}\right) = \frac{\rho \,.\, k_2^2}{2A}, \quad \text{where } A = \left(\frac{\tau_{w1} \,.\, r_1}{r_m^2 - r_1^2}\right) \tag{6}$$

With $\theta = (r/r_2)$, $\alpha = (r_2/r_1)$ and $\partial u/\partial r \simeq -\infty$ very near to the outer wall, equation (6) can be integrated to

$$\dot{u} = \frac{\sqrt{3} \,.\, u_{\tau 1} \,.\, \theta}{k_2 \,.\, \sqrt{(r_m^2 - r_1^2)} \,.\, \sqrt{\alpha}} \cdot \frac{1}{\sqrt{(1 - \theta^3)}}. \tag{7}$$

On integrating equation (7) and substituting the appropriate boundary conditions, an equation for the velocity defect is obtained as

$$\left(\frac{u_m - u}{u_{\tau 1}}\right) = \sqrt{\frac{3}{\alpha}} \cdot \frac{1}{k_2} \frac{1}{\sqrt{[\theta_m - (1/\alpha)^2]}} \int_{\theta_m}^{\theta} \frac{\theta}{\sqrt{(1 - \theta^3)}} \,\mathrm{d}\theta. \tag{8}$$

An alternative and more useful form of the last equation is possible using Lamb's result for r_m, and τ_{w2} from equation (5). Equation (8) is then transformed into

$$\left(\frac{u_m - u}{u_{\tau 2}}\right) = \frac{\sqrt{3}}{k_2} \cdot \frac{1}{\sqrt{1 - \theta_m^2}} \int_{\theta_m}^{\theta} \frac{\theta \,.\, \mathrm{d}\theta}{\sqrt{(1 - \theta^3)}} + b_2 \tag{9}$$

where b_2 serves to improve the agreement between the theoretical result and the experimental values.

Goldstein's tabulated values of the integral junction of equation (2) can be used to advantage here, because the integral in equation (9) can be written as

$$\int_{\theta_m}^{\theta} \frac{\theta}{\sqrt{(1 - \theta^3)}} \,.\, \mathrm{d}\theta = \int_{0}^{\theta} \frac{\theta}{\sqrt{(1 - \theta^3)}} \,.\, \mathrm{d}\theta - \int_{0}^{\theta_m} \frac{\theta}{\sqrt{(1 - \theta^3)}} \,.\, \mathrm{d}\theta \tag{10}$$

It is worth noting that the integrals on the right hand side of equation (10) are related to the Incomplete Beta Function*, $B_x(2/3, 1/2)$.

* The Incomplete Beta Function, $B_x(p, q)$, is defined as

$$B_x(p, q) = \int_0^x x^{p-1} \,.\, (1 - x)^{q-1} \,.\, \mathrm{d}x.$$

In any particular problem, the value of the last integral in equation (10) is constant, whereas the second integral is tabulated [1]. The compatability of the final result with the pipe result is to be noted; as $r_m \to 0$, $\theta_m \to 0$ and Equations (2) and (10) are then identical.

(ii) *Velocity distribution inside the radius of maximum velocity*

A similar analysis to that described in Section 4 (i) yields

$$\left(\frac{u_m - u}{u_{\tau 1}}\right) = \frac{\sqrt{3}}{k_1 \sqrt{(\psi_m^2 - 1)}} \cdot \int_{\psi}^{\psi_m} \frac{\psi}{\sqrt{(\psi^3 - 1)}} \,.\, \mathrm{d}\psi + b_1 \tag{11}$$

where $\psi = (r/r_1)$. k_1 and b_1 are chosen for the best fit of equation (11) to the inner region data.

5. COMPARISON BETWEEN THE THEORETICAL PREDICTION AND EXPERIMENTAL RESULTS IN ANNULAR FLOW

Figures 2(a) and 2(b) show a wide variety of experimental data for annular flow plotted in the usual manner, the Reynolds numbers and the radius ratios being as indicated.

A close examination of the points in Fig. 2(a) (outer region) will show that, with the exception of the data for $\alpha = 80{\cdot}72$, there is no detectable dependency on Reynolds number. The annulus, for which $\alpha = 80{\cdot}72$, shows [6] a radius of maximum velocity very much less than that predicted by Lamb [7] for laminar flow. The data for the more practical values of α are in very good agreement with the theoretical curve over the middle region, when $b_2 = -0{\cdot}8$ and $k_2 = +0{\cdot}148$.

The corresponding values for the inner region are plotted in Fig. 2(b). Here there is little effect of the Reynolds number but a marked dependency on the radius ratio. It is evident that it is impossible to correlate the inner velocities by a single equation, but equation (11) with $b_1 = -0{\cdot}8$ and $k_1 = +0{\cdot}148$ is shown for comparison purposes.

A possible reason for the discrepancy between theory and practice for the region inside

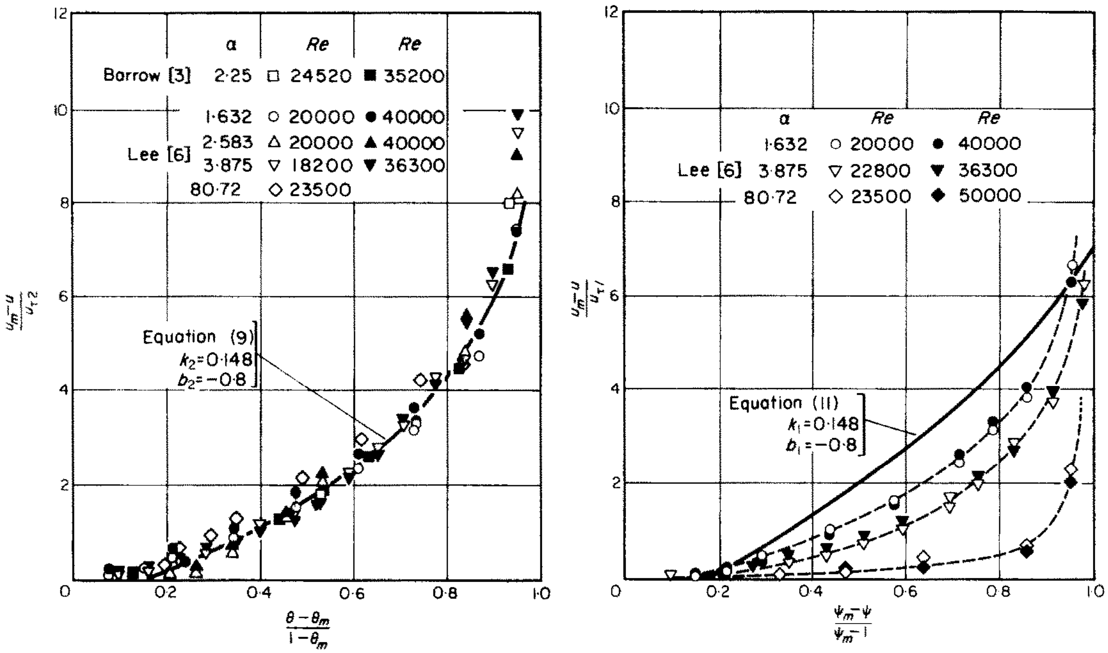

FIG. 2. The velocity distribution in turbulent flow in an annulus, and its correlation.
(a) Outside radius of maximum velocity. (b) Inside radius of maximum velocity.

r_m has already been given in the Section on Turbulence Characteristics. The assumption that $(\overline{u_1 v_1}/\overline{u_1^2})_1$ is constant is not fulfilled there, and the value of this ratio appears to be a function of both position and radius ratio. The excellent agreement between the theory and experiment for practical radius ratios outside the radius of maximum velocity is in accordance with the constancy of $(\overline{u_1 v_1}/\overline{u_1^2})_2$ in that that region.

6. CONCLUSIONS

It has been found that the use of a form of the similarity theory of Goldstein [1] is justified in predicting the velocity distribution outside the radius of maximum velocity in turbulent annular flow. An explanation for the difference between theory and experiment in the inner region has been given.

While the modified "laws of the wall" are of more direct use, the present analysis and considerations afford a deeper insight into the understanding and the prediction of the turbulent velocity field. There is accordingly a need for further detailed measurements of the nature and structure of turbulent flow in the annulus geometry. The position of the radius of maximum velocity can, for example, be measured accurately by direct experimentation but the location of the position of zero shear is as yet undetermined. We have found that the following expression correlates all the data for the radius of maximum velocity used in this paper:

$$r_m = r_1 \cdot \sqrt{\left(\frac{\alpha^2 - 1}{2 \ln \alpha}\right)} \cdot \left(\frac{1}{\alpha}\right)^{0\cdot027} \qquad (12)$$

(α not greater than 10)

Compared with the empirical correlation of Leung, Kays and Reynolds [12], equation (12) predicts a smaller deviation from Lamb's radius of maximum velocity.

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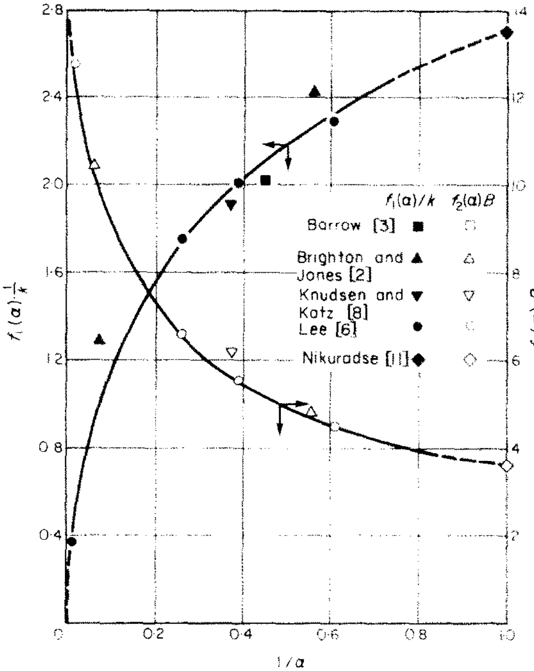

FIG. A1. The determination of the "constants" in the "modified law of wall" for the inner region of the annular flow.

APPENDIX

For engineering purposes, a modified "law of the wall" is adequate for the correlation of the velocity distribution inside the radius of maximum velocity, and here we derive an equation of the form

$$u^+ = \frac{f_1(\alpha)}{k} \ln y^+ + f_2(\alpha) \,.\, B \qquad \text{(A.1)}$$

To determine $f_1(\alpha)$ and $f_2(\alpha)$, a whole range of data [2, 3, 6, 8, 11] has been examined. It should be noted that outside r_m, the simple form of equation (A.1) viz. equation (3) suffices, $f_1(\alpha)$ and $f_2(\alpha)$ both having a value equal to unity. It is difficult to determine the best values of k and B; both Deissler's law [5] and the Prandtl–Nikuradse law fit the data well. The agreement between equation (3) and the data for the outer region of the annulus might be expected, because the ratio of the boundary layer thickness to outer wall radius is often much less than that for the pipe and consequently the lateral curvature effects are reduced. Inside r_m, however, there is a pronounced curvature effect of the inner wall, and $f_1(\alpha)$ and $f_2(\alpha)$ are found [6] to depend on α.

Figure A1 shows various values of $f_1(\alpha)/k$ and $f_2(\alpha).B$. The shapes of the curves are such that $f_1(\alpha)/k$ and $f_2(\alpha).B$ take the accepted values for symmetrical two dimensional flow when $(1/\alpha) \to 1$, which must be the case if the correlation is to be meaningful. The recommended equations are:

$$\left.\begin{array}{l} \text{(i) } f_1(\alpha)/k = 2\cdot7 \,.\, \left(\dfrac{1}{\alpha}\right)^{0\cdot353} \\ \text{and} \\ \text{(ii) } f_2(\alpha) \,.\, B = 3\cdot6 \,.\, \left(\dfrac{1}{\alpha}\right)^{-0\cdot439} \quad (\text{for } \alpha < 10) \end{array}\right\} \text{(A.2)}$$

It should be noted that in determining this correlation reference need not be made to the position of maximum velocity.

Résumé—La distribution de vitesse dans un écoulement turbulent entièrement développé à l'intérieur d'un tuyau annulaire est étudié en utilisant l'extension par Goldstein [1] de l'hypothèse de similitude de von Kármán. Le résultat thèorique est comparé avec les données expérimentales de plusieurs études. La validité de la théorie de la similitude dans l'étude de l'écoulement turbulent dans un tuyau annulaire est examiné en employant les mesures des caractéristiques de turbulence de Brighton et Jones [2]. Les résultats indiquent que l'adoption de la théorie est justifiée dans la région externe seulement de l'écoulement, c'est-à-dire en dehors du rayon de vitesse maximale. Une forme particulière de la théorie, que l'on avait trouvée [1] prédire avec précision la distribution de vitesse dans la géométrie plus simple d'un écoulement dans un tuyau circulaire, est employée dans l'étude du tuyau annulaire. La théorie de l'écoulement annulaire est telle que les résultats calculés pour le tuyau circulaire peuvent alors être utilisés avantageusement.

Les corrélations plus simples pour la distribution de vitesse de la forme de la "loi de la paroi" sont discutées brièvement, et l'insuffisance de ces corrélations dans la région interne de l'écoulement, c'est-à-dire à l'intérieur du rayon de vitesse maximale, est signalée. Enfin, une équation modifiée simple et de nature semi-empirique est présentée. Le résultat final pour la région interne est compatible avec les résultats pour les limites de la géométrie annulaire, par exemple le tuyau circulaire et la conduite à parois parallèles. On a remarqué que la distribution de vitesse dans la région externe du tuyau annulaire est décrite avec précision à l'aide de lois logarithmiques simples.

Zusammenfassung—Die Geschwindigkeitsverteilung bei voll ausgebildeter turbulenter Strömung im Ringspalt wird mit Hilfe der Erweiterung der von Kármánschen Ähnlichkeitshypothese nach Goldstein [1] untersucht. Das theoretische Ergebnis wird mit experimentellen Daten verschiedener Untersuchungen verglichen. Die Gültigkeit der Ähnlichkeitstheorie für turbulente Strömung im Ringspalt wird auf Grund der Messungen der Turbulenzcharakteristiken von Brighton und Jones [2] nachgeprüft. Die Ergebnisse zeigen, dass die Annahme der Theorie nur für den äusseren Bereich der Strömung, d.h. ausserhalb des Radius grösster Geschwindigkeit gerechtfertigt ist. Eine besondere Form der Theorie [1] nach der sich die Geschwindigkeitsverteilung in der einfacheren Rohrströmung genau bestimmen lässt wurde für die Ringspaltuntersuchung verwendet. Für die Analyse der Ringströmung werden die Rechenergebnisse für das Rohr vorteilhaft angewandt. Die einfacheren Korrelationen für die Geschwindigkeitsverteilung in Form des "Wandgesetzes" sind kurz diskutiert und auch ihre Unzulässigkeit für den inneren Bereich der Strömung, d.h. innerhalb des Radius grösster Geschwindigkeit dargelegt. Zur Vervollständigung wird eine einfache modifizierte Gleichung halbempirischer Natur angegeben. Das Endergebnis für den inneren Bereich ist verträglich mit den Ergebnissen für die Grenzen der Ringspaltgeometrie, nämlich dem Rohr und dem parallelwandigen Kanal. Es ist gezeigt, dass die Geschwindigkeitsverteilung im äusseren Bereich des Ringspaltes mit Hilfe einfacher logarithmischer Gesetze beschrieben werden kann.

Аннотация—Распределение скорости в полностью развитом турбулентном потоке в кольцевом канале исследовалось с помощью обобщенной Гольдштейном гипотезы подобия Кармана. Приведено сравнение теоретического результата с экспериментальными данными многих исследований. Справедливость теории подобия для изучения турбулентного потока в кольцевом канале проверяется, используя измерения турбулентных характеристик Брайтона и Джонса. Результаты показывают, что эта теория применима только для внешней области потока, т.е. за радиусом максимальной скорости. Для изучения процессов в кольцевом канале используется особая форма теории, которая, согласно работе [I], дает возможность предсказать распределение скорости потоков в трубах с более простой геометрией. Анализ потока в кольцевом канале построен так, что данные, полученные для трубы, можно с успехом использовать для других случаев.

Кратко рассматриваются более простые корреляции для распределения скорости в виде «закона стенки» и отмечается их недостаточность во внутренней области потока, т.е.в радиусе максимальной скорости. Для полноты приводится простое модифицированное полуэмпирическое уравнение. Конечный результат для внутренней области не противоречит результатам для предельных слуаев кольцевой геометрии, а именно для трубы и прямоточного канала. Отмечается, что расрпеделение скорости во внешней области кольцевого канала точно описывается с помощью простых логарифмических законов.